# 11.2

$$\vec{U} = \langle U_1, U_2, U_3 \rangle$$
$$\vec{V} = \langle V_1, V_2, V_3 \rangle$$

$$\vec{U} \cdot \vec{V} = U_1 V_1 + U_2 V_2 + U_3 V_3$$ ← DOT PRODUCT

SCALAR PRODUCT

OR

$$= \|\vec{U}\| \, \|\vec{V}\| \cos\theta$$


SAME INITIAL POINT

$\vec{U} \cdot \vec{V} > 0$ IF $\theta$ IS ACUTE

$< 0$ OBTUSE

$= 0$ $\vec{U} \perp \vec{V}$ OR $\vec{U} = \vec{0}$ OR $\vec{V} = \vec{0}$

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$$\vec{U} \cdot \vec{V} = \vec{V} \cdot \vec{U}$$

$$\vec{U} \cdot (\vec{V} + \vec{W}) = \vec{U} \cdot \vec{V} + \vec{U} \cdot \vec{W}$$

$$k(\vec{U} \cdot \vec{V}) = (k\vec{U}) \cdot \vec{V} = \vec{U} \cdot (k\vec{V})$$

$\underbrace{\underbrace{(\vec{U} \cdot \vec{V})}_{\text{}} \cdot \underbrace{\vec{W}}_{\text{}}}_{\text{SCALAR} \cdot \text{VECTOR}}$ $= \vec{U} \cdot (\vec{V} \cdot \vec{W})$ ? NO

SCALAR · VECTOR — ILLEGAL

NEITHER ~~OF~~ SIDE EXISTS

MULTIPLICATION DOES **NOT** DISTRIBUTE OVER MULTIPLICATION

$$\vec{U} \cdot \vec{U} = \|\vec{U}\| \|\vec{U}\| \cos 0 = \|\vec{U}\|^2$$

IF $\vec{u} = \langle -3,-2,4 \rangle$ AND $\vec{v} = \langle 1, c, -5 \rangle$

CRITICAL CONCEPTS

$\vec{u} \perp \vec{v} \longrightarrow \vec{u} \cdot \vec{v} = 0$

$\vec{u} \parallel \vec{v} \longrightarrow \vec{u} = k\vec{v}$

or

$\vec{v} = k\vec{u}$

(1) IF $\vec{u} \perp \vec{v}$, FIND $c$.

$$\vec{u} \cdot \vec{v} = 0$$

$$-3 - 2c - 20 = 0$$

$$-2c = 23$$

$$c = -\frac{23}{2}$$

(2) IF $\vec{w} = \langle a, b, 7 \rangle$ AND $\vec{u} \parallel \vec{w}$, FIND $a, b$.

$$\vec{w} = k\vec{u} \quad \text{or} \quad \vec{u} = k\vec{w}$$

$$\downarrow$$

$$\langle a,b,7 \rangle = k\langle -3,-2,4 \rangle$$

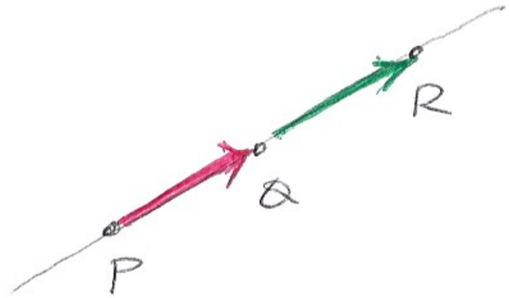
$$\langle a,b,7 \rangle = \langle -3k, -2k, 4k \rangle$$

$$a = -3k \longrightarrow a = -3 \cdot \frac{7}{4} = -\frac{21}{4}$$

$$b = -2k \longrightarrow b = -\cancel{2} \cdot \frac{7}{\cancel{4}_2} = -\frac{7}{2}$$

$$7 = 4k \longrightarrow k = \frac{7}{4}$$

ARE $P = (-1, 2, 3)$
$Q = (0, -1, 1)$
$R = (2, 1, -1)$ COLLINEAR?




IF YES, THEN $\overrightarrow{PQ} \parallel \overrightarrow{QR}$

$$\overrightarrow{PQ} = \langle 0-1, -1-2, 1-3 \rangle = \langle 1, -3, -2 \rangle$$
$$\overrightarrow{QR} = \langle 2-0, 1-1, -1-1 \rangle = \langle 2, 2, -2 \rangle$$

IF $\overrightarrow{PQ} \parallel \overrightarrow{QR}$

$$\overrightarrow{PQ} = k\overrightarrow{QR}$$
$$\langle 1, -3, -2 \rangle = k\langle 2, 2, -2 \rangle$$
$$= \langle 2k, 2k, -2k \rangle$$

$1 = 2k$ AND $-3 = 2k$ AND $-2 = -2k$

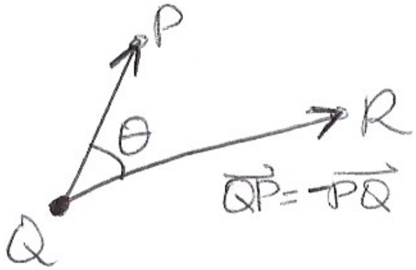
$k = \frac{1}{2}$ AND $k = -\frac{3}{2}$ } IMPOSSIBLE

$$\overrightarrow{PQ} \neq k\overrightarrow{QR}$$
$$\overrightarrow{PQ} \not\parallel \overrightarrow{QR}$$

P, Q, R ARE NOT COLLINEAR

(CONT'D)

FIND $\angle PQR$




$\overrightarrow{QP} = -\overrightarrow{PQ}$

$$\theta = \cos^{-1} \frac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{\|\overrightarrow{QP}\| \, \|\overrightarrow{QR}\|}$$

$$= \cos^{-1} \frac{\langle -1, 3, 2 \rangle \cdot \langle 2, 2, -2 \rangle}{\|\langle -1, 3, 2 \rangle\| \, \|\langle 2, 2, -2 \rangle\|}$$

$= \ldots$

# 11.3 CROSS PRODUCT or VECTOR PRODUCT

$$\vec{U} \times \vec{V} = \langle U_2V_3 - U_3V_2,\ U_3V_1 - U_1V_3,\ U_1V_2 - U_2V_1 \rangle$$

IF $\vec{U} = \langle U_1, U_2, U_3 \rangle$

AND $\vec{V} = \langle V_1, V_2, V_3 \rangle$

OR

$$\vec{U} \times \vec{V} = \underbrace{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \end{vmatrix}}_{\text{DETERMINANT}} \begin{matrix} \vec{i} & \vec{j} \\ U_1 & U_2 \\ V_1 & V_2 \end{matrix} = \begin{matrix} U_2V_3\vec{i} + U_3V_1\vec{j} + U_1V_2\vec{k} \\ -U_3V_2\vec{i} - U_1V_3\vec{j} - U_2V_1\vec{k} \end{matrix}$$

$$= (U_2V_3 - U_3V_2)\vec{i} + (U_3V_1 - U_1V_3)\vec{j} + (U_1V_2 - U_2V_1)\vec{k}$$

$$= \langle U_2V_3 - U_3V_2,\ U_3V_1 - U_1V_3,\ U_1V_2 - U_2V_1 \rangle$$

FIND $\underset{\vec{u}}{\langle 1,-1,2\rangle} \times \underset{\vec{v}}{\langle -2,3,1\rangle}$

$$\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & -1 & 2 & 1 & -1 \\ -2 & 3 & 1 & -2 & 3 \end{array}\right| = \begin{array}{l} -\vec{i} - 4\vec{j} + 3\vec{k} \\ -6\vec{i} - \vec{j} - 2\vec{k} \end{array}$$

$$= -7\vec{i} - 5\vec{j} + \vec{k} = \underbrace{\langle -7,-5,1\rangle}_{\vec{u}\times\vec{v}}$$

FIND $\langle -2,3,1\rangle \times \langle 1,-1,2\rangle$

$$\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -2 & 3 & 1 & -2 & 3 \\ 1 & -1 & 2 & 1 & -1 \end{array}\right| = \begin{array}{l} 6\vec{i} + \vec{j} + 2\vec{k} \\ +\vec{i} + 4\vec{j} - 3\vec{k} \end{array}$$

$$= 7\vec{i} + 5\vec{j} - \vec{k}$$

$$= \langle 7,5,-1\rangle$$

$$(\vec{u}\times\vec{v})\cdot\vec{u} = \langle -7,-5,1\rangle\cdot\langle 1,-1,2\rangle$$
$$= -7+5+2 = 0$$
$$(\vec{u}\times\vec{v})\cdot\vec{v} = \langle -7,-5,1\rangle\cdot\langle -2,3,1\rangle$$
$$= 14-15+1 = 0$$

$$\vec{u}\times\vec{v} \perp \vec{u},\vec{v}$$

$$\vec{u}\times\vec{v} \neq \vec{v}\times\vec{u}$$

$$\vec{u}\times\vec{v} = -(\vec{v}\times\vec{u})$$

CROSS PRODUCT IS <u>NOT</u> COMMUTATIVE

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin\theta$$ = AREA OF PARALLELOGRAM WITH $\vec{u}, \vec{v}$ AS ADJACENT SIDES